

Computing Temporal Defeasible Logics

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- Defeasible Logic is a simple and efficient (linear or polynomial time) non-monotonic formalism
- DL has been extended with time (temporalisation)
 - Natural representation of deadlines
 - Causality
 - Retroactivity

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Can we extend the good computational properties to temporal defeasible logic?

- Derive (plausible) conclusions with the minimum amount of information.
 - Definite conclusions
 - Defeasible conclusions
- Defeasible Theory
 - Facts
 - Strict Rules ($A_1, \dots, A_n \rightarrow B$)
 - Defeasible rules ($A_1, \dots, A_n \Rightarrow B$)
 - Defeaters ($A_1, \dots, A_n \rightsquigarrow B$)
 - Superiority relation over rules

Temporalised Defeasible Logic is an umbrella expression for a zoo of variants of logics.

- time points: A^t (A holds at time t)
- intervals: $A : [t_s, t_e]$ (A holds from t_s to t_e)
- durations: A^d (A holds for d time units)
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A temporalised defeasible theory

$$(F, R, >, \mathcal{T})$$

\mathcal{T} (discrete) total order of instants

- $+\partial p^t$: p has been defeasibly proved at time t (p holds at t)
- $-\partial p^t$: p has been defeasibly rejected at time t (p does not hold at t)

- propositions (literals) are associated with instants of time
 - C^t is **persistent** at time t , if C continues to hold after t unless some event occurs to terminate it.
 - C^t is **transient** at time t , if C is guaranteed to hold at time t only.

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- 1 Generate an argument for the persistent conclusion now using persistent rules.
 - Take a rule for the conclusion that is applicable now or
 - Show there is a time in the past where the persistent conclusion obtains.
- 2 Consider all possible counterarguments for the conclusion
 - Take all rules for its negation that obtain now
 - Take all rules for its negation that have obtained since the time in the past.
- 3 rebut the counterarguments
 - show that the rules have been discarded (not applicable or defeated).

Proving Conclusions in TDL





Warning: The presenter wishes to advise that the content of the next slide is restricted to a mathematical audience

If $P(n+1) = +\partial p^{t_p}$, then

1) $\exists r \in R_d^\pi[p^{t'_p}]$ such that

1) $\forall a^{t_a} \in A(r) : +\partial a^{t_a} \in P[1..n]$, and

2) $\forall s \in R[\sim p^{t_{\sim p}}]$ either

1) $\exists b^{t_b} \in A(s)$, $-\partial b^{t_b} \in P[1..n]$ or

2) $\exists w \in R[p^{t_{\sim p}}]$ such that

$\forall c^{t_c} \in A(w) : +\partial c^{t_c} \in P[1..n]$ and

$w \succ s$.

where $t'_p \leq t_{\sim p} \leq t_p$

Example



Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^\pi E^t$

$r_2 : B^t \Rightarrow^\pi \neg E^t$

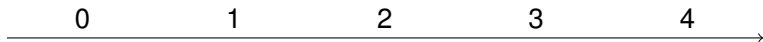
$r_3 : C^t \rightsquigarrow^\pi E^t$

$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$



Example



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Conclusions at time 0

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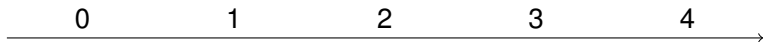
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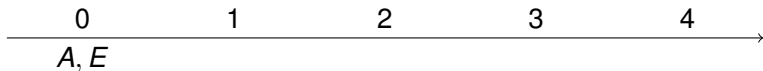
Conclusions at time 0

A, E using r_1 (E is persistent)

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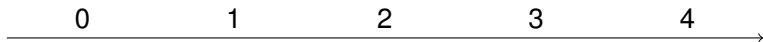
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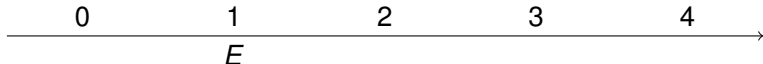
Conclusions at time 1

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Conclusions at time 1

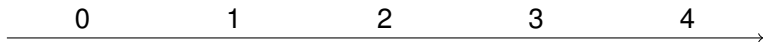
E

Conclusions at time 2

Superiority relation:

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$r_1 > r_4$



Example



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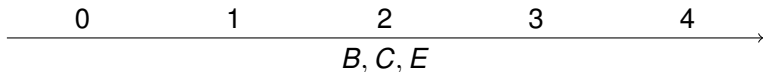
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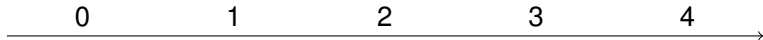
Conclusions at time 1

E

Conclusions at time 2

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Conclusions at time 3



Example



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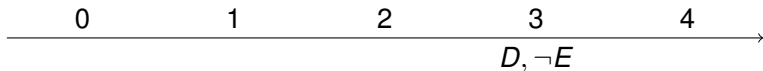
E

Conclusions at time 2

B, C, E

Conclusions at time 3

$D, \neg E$ using r_4



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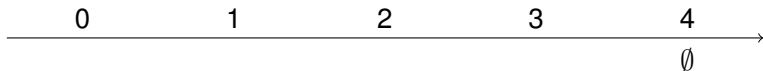
Conclusions at time 2

B, C, E

Conclusions at time 3

$D, \neg E$ using r_4

Conclusion at time 4



How not to compute conclusions



If all rules $a_1^{t_1}, \dots, a_n^{t_n} \Rightarrow p^t$ are such $\max(\{t_1, \dots, t_n\}) \leq t$.

- At time 0, consider the sub-theory restricted to the rules whose consequent is labelled by 0. Then use the basic algorithms for DL to compute the extension of the sub-theory at time 0.
- At time $n + 1$, consider the extension at time n . Then for each positive conclusion (i.e., conclusion whose proof tag is $+\partial$) p_i :
 - introduce a rule $r_{p_i}^n : \Rightarrow^{\tau} p_i$
 - introduce an instance of the superiority relation $r_{p_i}^n \prec s$ for each s such that $C(s) = \sim p_i^{n+1}$;
 - remove p_i^n from the body of rules where it occurs;

For each negative conclusion q_j remove rules where q_j appears in the body. Compute the extension for the sub-theory restricted to the rules whose consequent is labelled with $n + 1$.

Theorem

Let D be a theory in TDL. For $y \in \{\pi, \tau\}$:

- 1 If $D \vdash +\partial p^t$, then
 $D \cup \{r : p_1^{t_1}, \dots, p_n^{t_n}, p^t \Rightarrow^y q\} \equiv D \cup \{r : p_1^{t_1}, \dots, p_n^{t_n} \Rightarrow^y q\}.$
- 2 if $D \vdash -\partial p^t$, then $D \cup \{r : p_1^{t_1}, \dots, p_n^{t_n}, p^t \Rightarrow^y q\} \equiv D.$

Theorem

Let D be a TDL theory. If $r: \Rightarrow^x p^t \in R$ and $R[\sim p^t] \subseteq R_{\text{infd}}$, then $D \vdash +\partial p^t$ and $D \vdash -\partial \sim p^t$.

where $R_{\text{infd}} = \{r : \exists s, s \succ r, \text{ and } A(s) = \emptyset\}$.

Theorem

Let D be a TDL theory. Let $t_p = \min\{t : \exists r \in R[p^t]\}$, then $D \vdash -\partial p^{t'}$, $t' < t_p$.

- Conclusions are represented as $(l, [t, t'[]$
- For persistent conclusions expand the interval till the time of the next rule for the complementary literal
- When removing rules update the intervals of the already proved conclusions

Example



$$r: \Rightarrow^{\pi} p^1, \quad s: q^5 \Rightarrow^{\tau} \neg p^{10}, \quad v: \Rightarrow^{\pi} \neg p^{15}.$$

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Example



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$(+\partial p, [1, 15[)$

Theorem

*The extension of TDL can be computed in $O(S * |\mathcal{T}|)$, where S is the number of instances of literals occurring in the theory and \mathcal{T} is the set of distinct times in the theory.*

$$\begin{aligned} &\Rightarrow^{\pi} \textit{presentation}^{10:30} \\ &\rightsquigarrow^{\tau} \neg \textit{presentation}^{10:55} \\ \neg \textit{presentation}^{10:55} &\Rightarrow^{\pi} \textit{questions}^{10:55} \end{aligned}$$

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Questions?