Computing Temporal Defeasible Logics

Guido Governatori and Antonino Rotolo[1]

[1] CIRSFID, University of Bologna

RuleML 2013, Seattle, 14 July 2013

Australian Government
Department of Broadband, Communications and the Digital Economy
Australian Research Council

NICTA Funding and Supporting Members and Partners

Introduction

- Defeasible Logic is a simple and efficient (linear or polynomial time) non-monotonic formalism
- DL has been extended with time (temporalisation)
  - Natural representation of deadlines
  - Causality
  - Retroactivity

Introduction

- Defeasible Logic is a simple and efficient (linear or polynomial time) non-monotonic formalism
- DL has been extended with time (temporalisation)
  - Natural representation of deadlines
  - Causality
  - Retroactivity

Can we extend the good computational properties to temporal defeasible logic?

Basic(s) Defeasible Logic

- Derive (plausible) conclusions with the minimum amount of information.
  - Definite conclusions
  - Defeasible conclusions
- Defeasible Theory
  - Facts
  - Strict Rules ($A_1, \ldots, A_n \rightarrow B$)
  - Defeasible rules ($A_1, \ldots, A_n \Rightarrow B$)
  - Defeaters ($A_1, \ldots, A_n \rightsquigarrow B$)
  - Superiority relation over rules

Temporalised Defeasible Logic

Temporalised Defeasible Logic is an umbrella expression for a zoo of variants of logics.

- time points: A^t (A holds at time t)
- intervals: $A : [t_s, t_e]$ (A holds from t_s to t_e)
- durations: A^d (A holds for d time units)
- ...

Temporalised Defeasible Logic

Temporalised Defeasible Logic is an umbrella expression for a zoo of variants of logics.

- time points: A^t (A holds at time t)
- intervals: $A : [t_s, t_e]$ (A holds from t_s to t_e)
- durations: A^d (A holds for d time units)
- ...

A temporalised defeasible theory

$$(F, R, >, \mathcal{T})$$

$\mathcal{T}$ (discrete) total order of instants

Conclusion Types

- $+\partial p^t$: p has been defeasibly proved at time t (p holds at t)
- $-\partial p^t$: p has been defeasibly rejected at time t (p does not hold at t)

Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - C^t is persistent at time t, if C continues to hold after t unless some event occurs to terminate it.
  - C^t is transient at time t, if C is guaranteed to hold at time t only.

Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - C^t is persistent at time t, if C continues to hold after t unless some event occurs to terminate it.
  - C^t is transient at time t, if C is guaranteed to hold at time t only.
- partition the rules into persistent rules and transient rules

$$ClapHands^t \Rightarrow^\tau MakeSomeNoise^t$$

$$TearPaper^t \Rightarrow^\pi ShreddedPaper^t$$

Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - C^t is persistent at time t, if C continues to hold after t unless some event occurs to terminate it.
  - C^t is transient at time t, if C is guaranteed to hold at time t only.
- partition the rules into persistent rules and transient rules

$$ClapHands^t \Rightarrow^\tau MakeSomeNoise^t$$

$$TearPaper^t \Rightarrow^\pi ShreddedPaper^t$$

$$A_1^{t_1}, \ldots A_n^{t_n} \Rightarrow^x C^t$$

Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - C^t is persistent at time t, if C continues to hold after t unless some event occurs to terminate it.
  - C^t is transient at time t, if C is guaranteed to hold at time t only.
- partition the rules into persistent rules and transient rules

$$ClapHands^t \Rightarrow^\tau MakeSomeNoise^t$$

$$TearPaper^t \Rightarrow^\pi ShreddedPaper^t$$

$$A_1^{t_1}, \ldots A_n^{t_n} \Rightarrow^x C^t$$

no constraints over $t_1, \ldots, t_n$ and t.

Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - C^t is persistent at time t, if C continues to hold after t unless some event occurs to terminate it.
  - C^t is transient at time t, if C is guaranteed to hold at time t only.
- partition the rules into persistent rules and transient rules

$$ClapHands^t \Rightarrow^{\tau} MakeSomeNoise^t$$

$$TearPaper^t \Rightarrow^{\pi} ShreddedPaper^t$$

$$A_1^{t_1}, \ldots A_n^{t_n} \Rightarrow^{x} C^t$$

no constraints over $t_1, \ldots, t_n$ and t.

Proving Persistence

1. Generate an argument for the persistent conclusion now using persistent rules.
   - Take a rule for the conclusion that is applicable now or
   - Show there is a time in the past where the persistent conclusion obtains.
2. Consider all possible counterarguments for the conclusion
   - Take all rules for its negation that obtain now
   - Take all rules for its negation that have obtained since the time in the past.
3. rebut the counterarguments
   - show that the rules have been discarded (not applicable or defeated).

Proving Conclusions in TDL

MA

Proving Conclusions in TDL

MA

Warning: The presenter wishes to advise that the content of the next slide is restricted to a mathematical audience

Proving Conclusions in TDL

If $P(n+1) = +\partial p^{t_p}$, then
1) $\exists r \in R^{\pi}_{d}[p^{t'_p}]$ such that
  1) $\forall a^{t_a} \in A(r) : +\partial a^{t_a} \in P[1..n]$, and
  2) $\forall s \in R[\sim p^{t_{\sim p}}]$ either
    1) $\exists b^{t_b} \in A(s), -\partial b^{t_b} \in P[1..n]$ or
    2) $\exists w \in R[p^{t_{\sim p}}]$ such that
      $\forall c^{t_c} \in A(w) : +\partial c^{t_c} \in P[1..n]$ and
      $w \succ s$.

where $t'_p \leq t_{\sim p} \leq t_p$

Example

Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^\pi E^t$
$r_2 : B^t \Rightarrow^\pi \neg E^t$
$r_3 : C^t \leadsto^\pi E^t$
$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

$$\xrightarrow{\quad 0 \qquad 1 \qquad 2 \qquad 3 \qquad 4 \quad}$$

Example

Facts: A^0, B^2, C^2, D^3

Conclusions at time 0

Rules:
$$r_1 : A^t \Rightarrow^\pi E^t$$
$$r_2 : B^t \Rightarrow^\pi \neg E^t$$
$$r_3 : C^t \leadsto^\pi E^t$$
$$r_4 : D^t \Rightarrow^\tau \neg E^t$$

Superiority relation:

$$r_3 > r_2$$
$$r_1 > r_4$$

0 1 2 3 4

Example

Facts: A^0, B^2, C^2, D^3

Rules:

$$\begin{aligned} r_1 &: A^t \Rightarrow^{\pi} E^t \\ r_2 &: B^t \Rightarrow^{\pi} \neg E^t \\ r_3 &: C^t \leadsto^{\pi} E^t \\ r_4 &: D^t \Rightarrow^{\tau} \neg E^t \end{aligned}$$

Superiority relation:

$$r_3 > r_2$$
$$r_1 > r_4$$

Conclusions at time 0

A, E using r_1 (E is persistent)

0	1	2	3	4
A, E				

Example

Facts: A^0, B^2, C^2, D^3

Rules:
$$
\begin{aligned}
r_1 &: A^t \Rightarrow^\pi E^t \\
r_2 &: B^t \Rightarrow^\pi \neg E^t \\
r_3 &: C^t \leadsto^\pi E^t \\
r_4 &: D^t \Rightarrow^\tau \neg E^t
\end{aligned}
$$

Superiority relation:

$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
A, E using r_1 (E is persistent)
Conclusions at time 1

0 1 2 3 4

Example

Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$
$r_2 : B^t \Rightarrow^{\pi} \neg E^t$
$r_3 : C^t \leadsto^{\pi} E^t$
$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
A, E using r_1 (E is persistent)
Conclusions at time 1
E

0	1	2	3	4
	E			

Example

Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$
$r_2 : B^t \Rightarrow^{\pi} \neg E^t$
$r_3 : C^t \leadsto^{\pi} E^t$
$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
A, E using r_1 (E is persistent)
Conclusions at time 1
E
Conclusions at time 2

0 1 2 3 4

Example

Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$
$r_2 : B^t \Rightarrow^{\pi} \neg E^t$
$r_3 : C^t \leadsto^{\pi} E^t$
$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
A, E using r_1 (E is persistent)
Conclusions at time 1
E
Conclusions at time 2
B, C, E

0 1 2 3 4

B, C, E

Example

Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$

$r_2 : B^t \Rightarrow^{\pi} \neg E^t$

$r_3 : C^t \leadsto^{\pi} E^t$

$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$

Conclusions at time 0

A, E using r_1 (E is persistent)

Conclusions at time 1

E

Conclusions at time 2

B, C, E

Conclusions at time 3

0 1 2 3 4

Example

Facts: A^0, B^2, C^2, D^3

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$

$r_2 : B^t \Rightarrow^{\pi} \neg E^t$

$r_3 : C^t \leadsto^{\pi} E^t$

$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$

Conclusions at time 0

A, E using r_1 (E is persistent)

Conclusions at time 1

E

Conclusions at time 2

B, C, E

Conclusions at time 3

D, $\neg E$ using r_4

0 1 2 3 4

$D, \neg E$

Example

Facts: A^0, B^2, C^2, D^3

Rules:
$$
\begin{aligned}
r_1 &: A^t \Rightarrow^{\pi} E^t \\
r_2 &: B^t \Rightarrow^{\pi} \neg E^t \\
r_3 &: C^t \rightsquigarrow^{\pi} E^t \\
r_4 &: D^t \Rightarrow^{\tau} \neg E^t
\end{aligned}
$$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$

Conclusions at time 0

A, E using r_1 (E is persistent)

Conclusions at time 1

E

Conclusions at time 2

B, C, E

Conclusions at time 3

D, $\neg E$ using r_4

Conclusion at time 4

0	1	2	3	4
				$\emptyset$

How not to compute conclusions

If all rules $a_1^{t_1}, \ldots, a_n^{t_n} \Rightarrow p^t$ are such $\max(\{t_1, \ldots t_n\}) \leq t$.

- At time 0, consider the sub-theory restricted to the rules whose consequent is labelled by 0. Then use the basic algorithms for DL to compute the extension of the sub-theory at time 0.
- At time $n+1$, consider the extension at time n. Then for each positive conclusion (i.e., conclusion whose proof tag is $+\partial$) p_i:
  - introduce a rule $r^n_{p_i} : \Rightarrow^{\tau} p_i$
  - introduce an instance of the superiority relation $r^n_{p_i} \prec s$ for each s such that $C(s) = \sim p_i^{n+1}$;
  - remove p_i^n from the body of rules where it occurs;

  For each negative conclusion q_j remove rules where q_j appears in the body. Compute the extension for the sub-theory restricted to the rules whose consequent is labelled with $n+1$.

Transformations

Theorem

Let D be a theory in TDL. For $y \in \{\pi, \tau\}$:

1. *If $D \vdash +\partial p^t$, then*
   $D \cup \{r : p_1^{t_1}, \ldots, p_n^{t_n}, p^t \Rightarrow^y q\} \equiv D \cup \{r : p_1^{t_1}, \ldots, p_n^{t_n} \Rightarrow^y q\}$.
2. *if $D \vdash -\partial p^t$, then $D \cup \{r : p_1^{t_1}, \ldots, p_n^{t_n}, p^t \Rightarrow^y q\} \equiv D$.*

When is something provable

Theorem

Let D be a TDL theory. If $r: \Rightarrow^x p^t \in R$ and $R[\sim p^t] \subseteq R_{infd}$, then $D \vdash +\partial p^t$ and $D \vdash -\partial \sim p^t$.

where $R_{infd} = \{r : \exists s, s \succ r, \text{ and } A(s) = \emptyset\}$.

Theorem

Let D be a TDL theory. Let $t_p = \min\{t : \exists r \in R[p^t]\}$, then $D \vdash -\partial p^{t'}$, $t' < t_p$.

Computing the extension

- Conclusions are represented as $(l, [t, t'[)$
- For persistent conclusions expand the interval till the time of the next rule for the complementary literal
- When removing rules update the intervals of the already proved conclusions

Example

$$r: \Rightarrow^{\pi} p^{1}, \qquad s: q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v: \Rightarrow^{\pi} \neg p^{15}.$$

Example

$$r: \Rightarrow^{\pi} p^{1}, \qquad s: q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v: \Rightarrow^{\pi} \neg p^{15}.$$

$(+\partial p, [1, 10[), (+\partial \neg p, [15, \infty[)$

Example

$$r\colon \ \Rightarrow^{\pi} p^{1}, \qquad s\colon q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v\colon \ \Rightarrow^{\pi} \neg p^{15}.$$

$(+\partial p, [1, 10[), (+\partial\neg p, [15, \infty[)$
$(-\partial q, [0, \infty[)$ remove s

Example

$$r\colon \Rightarrow^{\pi} p^{1}, \qquad s\colon q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v\colon \Rightarrow^{\pi} \neg p^{15}.$$

$(+\partial p, [1, 10[), (+\partial\neg p, [15, \infty[)$
$(-\partial q, [0, \infty[)$ remove s
$(+\partial p, [1, 15[)$

Complexity

Theorem

The extension of TDL can be computed in $O(S * |\mathcal{T}|)$, where S is the number of instances of literals occurring in the theory an $\mathcal{T}$ is the set of distinct times in the theory.

From Theory to Practice

$$\Rightarrow^{\pi} \textit{presentation}^{10:30}$$

$$\leadsto^{\tau} \neg\textit{presentation}^{10:55}$$

$$\neg\textit{presentation}^{10:55} \Rightarrow^{\pi} \textit{questions}^{10:55}$$

From Theory to Practice

$$\Rightarrow^{\pi} \textit{presentation}^{10:30}$$

$$\rightsquigarrow^{\tau} \neg\textit{presentation}^{10:55}$$

$$\neg\textit{presentation}^{10:55} \Rightarrow^{\pi} \textit{questions}^{10:55}$$

Questions?