NICTA

# Computing Temporal Defeasible Logics

Guido Governatori and Antonino Rotolo[1]

[1] CIRSFID, University of Bologna

RuleML 2013, Seattle, 14 July 2013

Australian Government
Department of Broadband, Communications and the Digital Economy
Australian Research Council

NICTA Funding and Supporting Members and Partners

# Introduction

- Defeasible Logic is a simple and efficient (linear or polynomial time) non-monotonic formalism
- DL has been extended with time (temporalisation)
  - Natural representation of deadlines
  - Causality
  - Retroactivity

# Introduction

- Defeasible Logic is a simple and efficient (linear or polynomial time) non-monotonic formalism
- DL has been extended with time (temporalisation)
  - Natural representation of deadlines
  - Causality
  - Retroactivity

Can we extend the good computational properties to temporal defeasible logic?

# Basic(s) Defeasible Logic

- Derive (plausible) conclusions with the minimum amount of information.
  - Definite conclusions
  - Defeasible conclusions
- Defeasible Theory
  - Facts
  - Strict Rules ($A_1, \dots, A_n \to B$)
  - Defeasible rules ($A_1, \dots, A_n \Rightarrow B$)
  - Defeaters ($A_1, \dots, A_n \leadsto B$)
  - Superiority relation over rules

# Temporalised Defeasible Logic

Temporalised Defeasible Logic is an umbrella expression for a zoo of variants of logics.

- time points: $A^t$ ($A$ holds at time $t$)
- intervals: $A : [t_s, t_e]$ ($A$ holds from $t_s$ to $t_e$)
- durations: $A^d$ ($A$ holds for $d$ time units)
- ...

# Temporalised Defeasible Logic

Temporalised Defeasible Logic is an umbrella expression for a zoo of variants of logics.

- time points: $A^t$ ($A$ holds at time $t$)
- intervals: $A : [t_s, t_e]$ ($A$ holds from $t_s$ to $t_e$)
- durations: $A^d$ ($A$ holds for $d$ time units)
- ...

A temporalised defeasible theory

$$(F, R, >, \mathcal{T})$$

$\mathcal{T}$ (discrete) total order of instants

# Conclusion Types

- $+\partial p^t$: $p$ has been defeasibly proved at time $t$ ($p$ holds at $t$)
- $-\partial p^t$: $p$ has been defeasibly rejected at time $t$ ($p$ does not hold at $t$)

# Persistent and Transient Conclusions

NICTA

- propositions (literals) are associated with instants of time
  - $C^t$ is persistent at time $t$, if $C$ continues to hold after $t$ unless some event occurs to terminate it.
  - $C^t$ is transient at time $t$, if $C$ is guaranteed to hold at time $t$ only.

# Persistent and Transient Conclusions

NICTA

- propositions (literals) are associated with instants of time
  - $C^t$ is persistent at time $t$, if $C$ continues to hold after $t$ unless some event occurs to terminate it.
  - $C^t$ is transient at time $t$, if $C$ is guaranteed to hold at time $t$ only.
- partition the rules into persistent rules and transient rules

$$ClapHands^t \Rightarrow^\tau MakeSomeNoise^t$$

$$TearPaper^t \Rightarrow^\pi ShreddedPaper^t$$

# Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - $C^t$ is persistent at time $t$, if $C$ continues to hold after $t$ unless some event occurs to terminate it.
  - $C^t$ is transient at time $t$, if $C$ is guaranteed to hold at time $t$ only.
- partition the rules into persistent rules and transient rules

$$ClapHands^t \Rightarrow^\tau MakeSomeNoise^t$$

$$TearPaper^t \Rightarrow^\pi ShreddedPaper^t$$

$$A_1^{t_1}, \ldots A_n^{t_n} \Rightarrow^x C^t$$

# Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - $C^t$ is persistent at time $t$, if $C$ continues to hold after $t$ unless some event occurs to terminate it.
  - $C^t$ is transient at time $t$, if $C$ is guaranteed to hold at time $t$ only.
- partition the rules into persistent rules and transient rules

$$\mathit{ClapHands}^t \Rightarrow^{\tau} \mathit{MakeSomeNoise}^t$$

$$\mathit{TearPaper}^t \Rightarrow^{\pi} \mathit{ShreddedPaper}^t$$

$$A_1^{t_1}, \ldots A_n^{t_n} \Rightarrow^{x} C^t$$

no constraints over $t_1, \ldots, t_n$ and $t$.

# Persistent and Transient Conclusions

- propositions (literals) are associated with instants of time
  - $C^t$ is persistent at time $t$, if $C$ continues to hold after $t$ unless some event occurs to terminate it.
  - $C^t$ is transient at time $t$, if $C$ is guaranteed to hold at time $t$ only.
- partition the rules into persistent rules and transient rules

$$\mathit{ClapHands}^t \Rightarrow^{\tau} \mathit{MakeSomeNoise}^t$$

$$\mathit{TearPaper}^t \Rightarrow^{\pi} \mathit{ShreddedPaper}^t$$

$$A_1^{t_1}, \ldots A_n^{t_n} \Rightarrow^{x} C^t$$

no constraints over $t_1, \ldots, t_n$ and $t$.

# Proving Persistence

1. Generate an argument for the persistent conclusion now using persistent rules.
   - Take a rule for the conclusion that is applicable now or
   - Show there is a time in the past where the persistent conclusion obtains.
2. Consider all possible counterarguments for the conclusion
   - Take all rules for its negation that obtain now
   - Take all rules for its negation that have obtained since the time in the past.
3. rebut the counterarguments
   - show that the rules have been discarded (not applicable or defeated).

# Proving Conclusions in TDL

MA

# Proving Conclusions in TDL

**MA**

Warning: The presenter wishes to advise that the content of the next slide is restricted to a mathematical audience

# Proving Conclusions in TDL

If $P(n+1) = +\partial p^{t_p}$, then

1) $\exists r \in R_d^{\pi}[p^{t'_p}]$ such that
    1) $\forall a^{t_a} \in A(r) : +\partial a^{t_a} \in P[1..n]$, and
    2) $\forall s \in R[\sim p^{t_{\sim p}}]$ either
        1) $\exists b^{t_b} \in A(s), -\partial b^{t_b} \in P[1..n]$ or
        2) $\exists w \in R[p^{t_{\sim p}}]$ such that
           $\forall c^{t_c} \in A(w) : +\partial c^{t_c} \in P[1..n]$ and
           $w \succ s$.

where $t'_p \leq t_{\sim p} \leq t_p$

# Example

NICTA

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules:
$$
\begin{aligned}
r_1 &: A^t \Rightarrow^{\pi} E^t\\
r_2 &: B^t \Rightarrow^{\pi} \neg E^t\\
r_3 &: C^t \rightsquigarrow^{\pi} E^t\\
r_4 &: D^t \Rightarrow^{\tau} \neg E^t
\end{aligned}
$$

Superiority relation:

$$r_3 > r_2$$
$$r_1 > r_4$$

0 1 2 3 4

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules:
$$
\begin{aligned}
r_1 &: A^t \Rightarrow^{\pi} E^t \\
r_2 &: B^t \Rightarrow^{\pi} \neg E^t \\
r_3 &: C^t \leadsto^{\pi} E^t \\
r_4 &: D^t \Rightarrow^{\tau} \neg E^t
\end{aligned}
$$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$

Conclusions at time 0

0 1 2 3 4

# Example

Facts: $A^0, B^2, C^2, D^3$

Rules: $r_1 : A^t \Rightarrow^\pi E^t$
$r_2 : B^t \Rightarrow^\pi \neg E^t$
$r_3 : C^t \leadsto^\pi E^t$
$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
$A, E$ using $r_1$ ($E$ is persistent)

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $A, E$ | | | | |

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules: $r_1 : A^t \Rightarrow^\pi E^t$
$r_2 : B^t \Rightarrow^\pi \neg E^t$
$r_3 : C^t \leadsto^\pi E^t$
$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
$A$, $E$ using $r_1$ ($E$ is persistent)
Conclusions at time 1

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules:
$$
\begin{aligned}
r_1 &: A^t \Rightarrow^{\pi} E^t \\
r_2 &: B^t \Rightarrow^{\pi} \neg E^t \\
r_3 &: C^t \rightsquigarrow^{\pi} E^t \\
r_4 &: D^t \Rightarrow^{\tau} \neg E^t
\end{aligned}
$$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$

Conclusions at time 0

$A$, $E$ using $r_1$ ($E$ is persistent)

Conclusions at time 1

$E$

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | $E$ | | | |

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules: $r_1 : A^t \Rightarrow^\pi E^t$

$r_2 : B^t \Rightarrow^\pi \neg E^t$

$r_3 : C^t \rightsquigarrow^\pi E^t$

$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:

$r_3 > r_2$

$r_1 > r_4$

Conclusions at time 0

$A$, $E$ using $r_1$ ($E$ is persistent)

Conclusions at time 1

$E$

Conclusions at time 2

0 1 2 3 4

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$
$r_2 : B^t \Rightarrow^{\pi} \neg E^t$
$r_3 : C^t \leadsto^{\pi} E^t$
$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
$A$, $E$ using $r_1$ ($E$ is persistent)
Conclusions at time 1
$E$
Conclusions at time 2
$B$, $C$, $E$

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \hline & & B, C, E & & \end{array}$$

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules: $r_1 : A^t \Rightarrow^\pi E^t$
$r_2 : B^t \Rightarrow^\pi \neg E^t$
$r_3 : C^t \leadsto^\pi E^t$
$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
$A$, $E$ using $r_1$ ($E$ is persistent)
Conclusions at time 1
$E$
Conclusions at time 2
$B$, $C$, $E$
Conclusions at time 3

$$\xrightarrow{\quad 0 \qquad\qquad 1 \qquad\qquad 2 \qquad\qquad 3 \qquad\qquad 4 \quad}$$

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules: $r_1 : A^t \Rightarrow^{\pi} E^t$
$r_2 : B^t \Rightarrow^{\pi} \neg E^t$
$r_3 : C^t \rightsquigarrow^{\pi} E^t$
$r_4 : D^t \Rightarrow^{\tau} \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
$A$, $E$ using $r_1$ ($E$ is persistent)
Conclusions at time 1
$E$
Conclusions at time 2
$B$, $C$, $E$
Conclusions at time 3
$D$, $\neg E$ using $r_4$

0 1 2 3 4

$D, \neg E$

# Example

Facts: $A^0$, $B^2$, $C^2$, $D^3$

Rules:
$r_1 : A^t \Rightarrow^\pi E^t$
$r_2 : B^t \Rightarrow^\pi \neg E^t$
$r_3 : C^t \leadsto^\pi E^t$
$r_4 : D^t \Rightarrow^\tau \neg E^t$

Superiority relation:
$r_3 > r_2$
$r_1 > r_4$

Conclusions at time 0
$A$, $E$ using $r_1$ ($E$ is persistent)
Conclusions at time 1
$E$
Conclusions at time 2
$B$, $C$, $E$
Conclusions at time 3
$D$, $\neg E$ using $r_4$
Conclusion at time 4

0 1 2 3 4

$\emptyset$

# How not to compute conclusions

NICTA

If all rules $a_1^{t_1}, \ldots, a_n^{t_n} \Rightarrow p^t$ are such $\max(\{t_1, \ldots t_n\}) \leq t$.

- At time 0, consider the sub-theory restricted to the rules whose consequent is labelled by 0. Then use the basic algorithms for DL to compute the extension of the sub-theory at time 0.
- At time $n + 1$, consider the extension at time $n$. Then for each positive conclusion (i.e., conclusion whose proof tag is $+\partial$) $p_i$:
  - introduce a rule $r^n_{p_i} : \Rightarrow^{\tau} p_i$
  - introduce an instance of the superiority relation $r^n_{p_i} \prec s$ for each $s$ such that $C(s) = {\sim}p_i^{n+1}$;
  - remove $p_i^n$ from the body of rules where it occurs;

  For each negative conclusion $q_j$ remove rules where $q_j$ appears in the body. Compute the extension for the sub-theory restricted to the rules whose consequent is labelled with $n + 1$.

# Transformations

### Theorem

*Let D be a theory in TDL. For $y \in \{\pi, \tau\}$:*

1. *If* $D \vdash +\partial p^t$, *then* $D \cup \{r : p_1^{t_1}, \ldots, p_n^{t_n}, p^t \Rightarrow^y q\} \equiv D \cup \{r : p_1^{t_1}, \ldots, p_n^{t_n} \Rightarrow^y q\}$.
2. *if* $D \vdash -\partial p^t$, *then* $D \cup \{r : p_1^{t_1}, \ldots, p_n^{t_n}, p^t \Rightarrow^y q\} \equiv D$.

# When is something provable

**Theorem**

*Let $D$ be a TDL theory. If $r\colon \Rightarrow^x p^t \in R$ and $R[\sim p^t] \subseteq R_{infd}$, then $D \vdash +\partial p^t$ and $D \vdash -\partial \sim p^t$.*

where $R_{infd} = \{r : \exists s, s \succ r, \text{ and } A(s) = \emptyset\}$.

**Theorem**

*Let $D$ be a TDL theory. Let $t_p = \min\{t\colon \exists r \in R[p^t]\}$, then $D \vdash -\partial p^{t'}$, $t' < t_p$.*

# Computing the extension

- Conclusions are represented as $(l, [t, t'[)$
- For persistent conclusions expand the interval till the time of the next rule for the complementary literal
- When removing rules update the intervals of the already proved conclusions

# Example

$$r: \Rightarrow^{\pi} p^{1}, \qquad s: q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v: \Rightarrow^{\pi} \neg p^{15}.$$

# Example

$$r: \ \Rightarrow^{\pi} p^{1}, \qquad s: q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v: \ \Rightarrow^{\pi} \neg p^{15}.$$

$(+\partial p, [1, 10[), (+\partial\neg p, [15, \infty[)$

# Example

$$r\colon \Rightarrow^{\pi} p^{1}, \qquad s\colon q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v\colon \Rightarrow^{\pi} \neg p^{15}.$$

$(+\partial p, [1, 10[), (+\partial\neg p, [15, \infty[)$
$(-\partial q, [0, \infty[)$ remove $s$

# Example

$$r: \Rightarrow^{\pi} p^{1}, \qquad s: q^{5} \Rightarrow^{\tau} \neg p^{10}, \qquad v: \Rightarrow^{\pi} \neg p^{15}.$$

$(+\partial p, [1, 10[), (+\partial \neg p, [15, \infty[)$
$(-\partial q, [0, \infty[)$ remove $s$
$(+\partial p, [1, 15[)$

## Complexity

**Theorem**

*The extension of TDL can be computed in $O(S * |\mathcal{T}|)$, where S is the number of instances of literals occurring in the theory an $\mathcal{T}$ is the set of distinct times in the theory.*

# From Theory to Practice

$$\Rightarrow^{\pi} \textit{presentation}^{10:30}$$

$$\leadsto^{\tau} \neg\textit{presentation}^{10:55}$$

$$\neg\textit{presentation}^{10:55} \Rightarrow^{\pi} \textit{questions}^{10:55}$$

# From Theory to Practice

$$\Rightarrow^{\pi} \textit{presentation}^{10:30}$$

$$\rightsquigarrow^{\tau} \neg\textit{presentation}^{10:55}$$

$$\neg\textit{presentation}^{10:55} \Rightarrow^{\pi} \textit{questions}^{10:55}$$

# Questions?